

Plane waves in dissipative transversely isotropic media

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In a perfectly elastic (non-dissipative) transversely isotropic (TI) medium (where “transverse” refers to directions normal to the depth (z) axis), phase velocities for plane SH waves vary with the direction of propagation, changing monotonically from a minimum (or maximum) in the vertical direction to a maximum (or minimum) in the horizontal direction. However, if the medium is also dissipative, then plane waves are generally inhomogeneous, and there is a range of directions in which the phase velocity drops to anomalously low values for waves with moderately high degrees of inhomogeneity, and in which waves with high degrees of inhomogeneity cannot propagate at all. The latter counter-intuitive result occurs because there is no physically acceptable solution to the dispersion relation. This curious result does not occur in a non-dissipative TI medium or in a dissipative isotropic medium. Fortunately, these waves do not appear in the plane wave superpositions corresponding to spherical or cylindrical waves, because they do not correspond to any points in the horizontal slowness plane. Plane waves of the former type though, i.e., with anomalously low phase speeds, could occur under the right conditions and cause a number of anomalous wave propagation effects.

Plane wave superpositions giving cylindrical or spherical waves in a dissipative TI medium contain inhomogeneous waves whose amplitudes grow in the direction of phase propagation, with this direction pointing away from the observation point. However, the energy in these waves does travel toward the observation point, and their amplitudes decay in the direction of energy propagation.

A dissipative TI medium also exhibits absorption anisotropy, i.e., the Q of a plane wave varies with its direction of propagation. Q is commonly defined as the ratio of the real and imaginary parts of the complex modulus. However, this definition leads to a generally unreliable measure of the amount of dissipation occurring in the propagation of an inhomogeneous plane wave. Again, this unreliability does not occur for dissipative isotropic or non-dissipative TI media. The more fundamental definition, based on the ratio of mean energy per cycle to energy loss per cycle gives reasonable results, and should be used to study Q anisotropy.

The computation of q_{SH} and q_P - q_{SV} reflection and transmission coefficients for an interface between two dissipative TI media requires knowledge of both the horizontal and vertical components of the slowness vectors of the incident and scattered waves. To obtain the correct value of the coefficient for a given scattered wave, the correct values (the values satisfying Fermat’s principle) of the incident propagation and attenuation angles must be specified. This is most easily done by using the correct complex value of the horizontal slowness, rather than dealing with propagation and attenuation angles. The vertical slownesses for the incident and scattered waves can be expressed in closed form, although the expression is much more complicated for q_P - q_{SV} waves than for q_{SH} waves.