Seismic reflection near critical angles

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The solution to Cagniard's problem, the response of an interface separating two solid, semi-infinite, homogeneous, isotropic media to an impulsive point source, has been extensively studied. The exact solution can be cast as an integral over ray parameter in the temporal frequency domain (see, for example, Aki and Richards, 1980). The various waves which comprise the full response, such as the reflected or head waves, manifest themselves as crucial points (i.e. saddle points and branch points) in the complex ray parameter plane. The high frequency steepest descents approximation of this integral leads to the usual expressions of geometrical ray theory. However, it is well known that these expressions fail in the vicinity of the critical angle, where a saddle point approaches a branch point in the complex plane. In this region the head wave interferes with the reflected wave and amplitudes become frequency dependent. Accurate approximations can be realized by constructing uniform asymptotic expansions (Bleistein, 1975), which is the method chosen by Cerveny (1971). These expressions are uniform in the sense that they are valid for angles both near and far from the critical angle. There is a trade-off in simplicity for accuracy, however, as the uniform approximations require the evaluation of a special function (in this case the Parabolic Cylinder Function).

The uniform solution proposed by Cerveny proves to be itself inadequate for strong interfaces, that is, ones across which the velocity contrast is large (for typical values of Poisson's ratio). This is due to the presence of poles in the integrand which lie on non-physical Riemann sheets (Gilbert and Laster, 1962) which can still influence the amplitudes of other waves (Chapman, 1972). These poles lie close to the branch point in question for strong velocity contrasts. The result is that the 'uniform' solution no longer accurately describes amplitudes.

A true uniform expansion must account for the poles in the integrand. We show one way to construct such an expansion, which requires the use of up to seven special functions. The result can still be less computationally intensive than employing numerical methods to evaluate the original exact integral. Depending upon the location of the poles in the complex plane, the result can often be simplified.

References

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