DISCUSSIONS

ERRORS IN THE USE AND PRESENTATION OF AN EMPIRICAL FORMULA FOR THE SETTLING VELOCITY OF SPHERES: DISCUSSION OF A PAPER BY RONALD J. GIBBS, MARTIN D. MATTHEWS, AND DAVID A. LINK

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A recent paper by Gibbs et al. (1971) misused an empirical formula given by Janke (1965) by applying it for comparative purposes over a domain for which it was explicitly not defined. Janke’s paper stated in its abstract (p. 749) that it was “A formula for the unhindered settling velocity of spheres with Reynolds’ numbers from 0.100 to about 100,000...” and again, in the first paragraph, “A formula, applicable to single spheres in a fluid where the effects of container boundaries are negligible, was developed for Reynolds’ numbers from the Stokes domain to about 100,000.” These Reynolds’ numbers correspond approximately to those for quartz spheres with diameters between 50 and 62,000 microns settling in water at 20°C., yet, in two places in the text of the Gibbs’ paper (pp. 7 and 12) they gave percentage discrepancies obtained by using Janke’s formula well below the range for which it was intended (one, ten and twenty microns). Janke’s formula was originally developed as part of a shape and settling velocity study because of the need for a formula that gave an estimate of the settling velocities of coarse particles, and, as such, it worked well as a base for developing the settling velocity relations for many non-spherical particles (Janke, 1966). At the time, a formula similar to that of Rubey (1933), which is basically the form presented by Gibbs et al., was tried and abandoned because it did not work sufficiently well with the data available over the coarse particle range, and because it was desirable to use a formula with a flow-regime related parameter (N in the formula below).

Since Gibbs et al. were primarily concerned with developing a formula for sedimentation tube studies, their formula is reasonably confined to Reynolds’ numbers of about 4000 or less, which correspond to those of quartz spheres settling in water at 20°C. with diameters of about 6000 microns and smaller, but to state (p. 12) “In addition, Janke’s equation is difficult to use and cannot be combined with Stokes’ equation without producing a discontinuity in the data (see fig. 1).” is to overstate their case and to unnecessarily depreciate Janke’s equation. With regard to the continuity objection, Janke’s equation was designed to give velocity values slightly lower than Stokes’ Law values, but within about 1%, at Reynolds’ numbers of 0.100 (equivalent to diameters slightly below 50 microns for quartz spheres settling at 20°C.). Below this Reynolds’ number, Stokes’ Law was assumed sufficient. If needed, Janke’s formula can be extended down to Reynolds’ numbers of 0.004 or less by adding a percentage correction given by P = -19.4 Log 10 Rn, where Rn is the Reynolds’ number calculated from Janke’s formula; however, it seems unnecessary to attempt to duplicate the values given by Stokes’ Law below Reynolds’ numbers of about 0.100, or to fit to them with better than about 1% precision.

With regard to the objection of difficulty of use, one could just as well adopt the viewpoint that for computing settling velocity, the formula of Gibbs et al. is the more difficult, since its four fitted constants with five significant figures each make it a practical necessity to use a calculator for repetitive multiplications, whereas Janke’s formula can be applied by using a slide rule and occasional reference to a 4-place log table. A convenient form of Janke’s formula is given below.

\[ R_n = \left(\frac{1}{21.2}\right)^{1.0N} \]

\[ N = \text{antilog} \left[ (0.0293 \log j - 7.85(10^{-4})(\log j)^2 + 0.008) \right] \]

\[ j = 4Dg\delta^2/3\nu^2 \] (dimensionless), \( D = (\text{density solid/density liquid}) - 1 \), \( g \) is the acceleration in units of the earth’s gravity field, \( \nu \) is the kinematic viscosity, and \( d \) is the diameter of the settling sphere. If a computer is used, there is no significant difference in the difficulty of use for either formula. Janke’s formula would, for

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